lowing for the influence of a single pore in coordinates X_i in the X_1 direction; $\varkappa''_{1\mathrm{eff}}$, effective thermal conductivity allowing for the influence of a single pore in coordinates x_i in the x_1 direction.

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DETERMINATION OF THE COEFFICIENT OF THERMAL CONDUCTIVITY BY TWO-POINT PROBING OF THE SPECIMEN SURFACE

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A comparison method of determining the coefficient of thermal conductivity which permits direct measurement on specimens of arbitrary geometry without their destruction is elucidated. Experimental results on realization of the method are presented.

Methods to determine the coefficient of thermal conductivity, based on surface heat probing of the specimens, are of great practical interest. The main advantage of such methods is the possibility of conducting measurements on specimens of arbitrary geometry, for example, on fabricated items, without their destruction.

A number of instruments which solve this problem to some extent is described in [1]. Underlying the instruments is the principle of point heat probing of the specimen surface and recording the temperature difference at two points of the probe, which characterizes the heat exchange between the probe and the specimen across the zone of their continuity in an almost stationary mode. The coefficient of thermal conductivity is determined by a comparison with the results of similar meausrements on standard specimens with a known thermal conductivity. Hence, such instruments have been called thermal comparators. One of the most successful, which yields the possibility of reading the coefficient of thermal conductivity directly on the scale of a recording device, is the thermal comparator consisting of a bulk Constantan module and a thin rod standing off therefrom, whose end is in thermal contact with the specimen surface. A measure of the thermal conductivity is the temperature difference between the end of the rod in contact with the specimen and a preheated Constantan module at a higher temperature compared to the specimen temperature, recorded by using a differential thermocouple in the steady-state mode.

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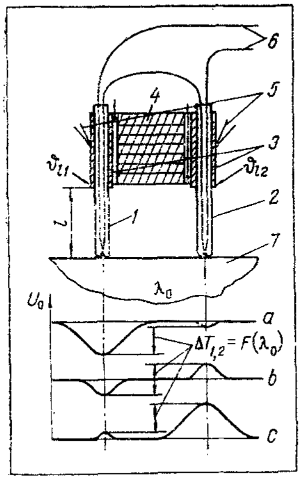

Fig. 1. Thermal comparator with two-point probing: 1, 2) heat probes; 3) copper plates; 4) thermoelectric battery; 5, 6) differential thermocouples: 7) specimen being tested; a) $U_0>\vartheta_{l2}>\vartheta_{l1}$; b) $\vartheta_{l2}>U_0>\vartheta_{l1}$; c) $\vartheta_{l2}>\vartheta_{l1}>U_0$.

However, a direct reading of the thermal conductivity on the instrument scale during measurements using such a comparator is possible only if not only the temperature difference between the Constantan module and the specimens, but also the temperature of the specimens being tested, which should equal the temperature standard specimens had when the instrument was calibrated, is kept identical from measurement to measurement. The inconsistency between the test conditions and the calibration conditions results in significant methodological errors.

A method affording the possibility of obtaining confident results under real conditions even for specimen temperatures differing from the temperature of the ambient medium while the instrument calibration is conducted at one, for example, room temperature is elucidated below. In contrast to the known method, it is based on two-point thermal probing of the surface by the two heat probes 1 and 2 (Fig. 1) in the shape of rods. By using the semiconductor thermoelectric battery 4 a constant temperature difference is maintained between the upper ends of the heat probes, which is established by the signal of the differential thermocouple 5. A measure of the thermal conductivity is the thermal emf of the differential thermocouple 6, which measures the temperature difference between the low ends of the heat probes in thermal contact with the specimen surface.

The expression [2]

$$\vartheta(x)=\frac{\vartheta_{01}\,\mathrm{sh}\,k(l-x)+\vartheta_{l1}\,\mathrm{sh}\,kx}{\mathrm{sh}\,kl} \tag{1}$$

is valid in the stationary mode for the temperature distribution along one of the heat probes, considered as a thin rod whose upper end ($x = l$) is maintained at the constant excess temperature ϑ_{l1} while the lower ($x = 0$) is in contact with the semibounded specimen and has some constant excess temperature ϑ_{01}.

The heat flux through the contact is

$$Q=-\lambda_m\frac{\partial\vartheta(x)}{\partial x}\Big|_{x=0}s=\lambda_m ks\,\frac{\vartheta_{01}\,\mathrm{ch}\,kl-\vartheta_{l1}}{\mathrm{sh}\,kl} \tag{2}$$

Let us assume that thermal contact between the heat probe and the specimen occurs within the limits of a circle of radius R; the temperature of the heat probe sensor equals the mean temperature of the specimen surface in this circle; there is no heat exchange between the specimen with the heat probe and the ambient medium within the limits of this circle. Then according to [3], the heat flux for the semibounded specimen is

$$Q=\frac{3\pi^2}{8}R\lambda_0[U_0-U_{av}(R,\ 0)]=\frac{3\pi^2}{8}R\lambda_0(U_0-\vartheta_{01}). \tag{3}$$

Equating (2) and (3) and solving for ϑ_{01}, we obtain

$$\vartheta_{01}=\frac{3\pi^2R\lambda_0U_0\,\mathrm{sh}\,kl+8\lambda_m ks\vartheta_{l1}}{3\pi^2R\lambda_0\,\mathrm{sh}\,kl+8\lambda_m ks\,\mathrm{ch}\,kl}. \tag{4}$$

It should be noted that this expression is valid for the temperature of the lower end of the rod even in the thermal comparator described in [1], in which the temperature ϑ_{l1} is maintained constant during the test because of the bulk Constantan module. The following expression obtained from (4):

$$\vartheta_{l1} - \vartheta_{01} = \frac{(\vartheta_{l1} - U_0)\,3\pi^2 R\lambda_0 \operatorname{sh} kl + 8\vartheta_{l1}\lambda_m ks(\operatorname{ch} kl - 1)}{3\pi^2 R\lambda_0 \operatorname{sh} kl + 8\lambda_m ks \operatorname{ch} kl}. \tag{5}$$

will hence be valid for the temperature difference being recorded between the end of the rod and the Constantan module. It is seen from this expression that both the temperature difference $\vartheta_{l1} - U_0$ and the temperature ϑ_{l1}, and, therefore, U_0, must be kept constant from test to test in order to compare the thermal conductivities of the standard and investigated specimens in a single-point probing. Only in this case can λ_0 be considered the single variable. To comply with these conditions, the instrument calibration and the measurements are ordinarily carried out at a specimen temperature equal to the temperature of the ambient medium ($U_0 = 0$) and with the excess temperature of the Constantan module ϑ_{l1} maintained constant. Measurements on specimens with a temperature different from the temperature of the ambient medium do not yield confident results.

From (4), the equality

$$\Delta T_{1,2} = \vartheta_{02} - \vartheta_{01} = \frac{8(\vartheta_{l2} - \vartheta_{l1})\lambda_m ks}{8\lambda_m ks \operatorname{ch} kl + 3\pi^2 R\lambda_0 \operatorname{sh} kl} \tag{6}$$

is valid for two-point heat probing by using two identical heat probes whose upper ends are kept at different temperatures ϑ_{l1} and ϑ_{l2} for a temperature difference at the lower ends which are in contact with the specimen.

If the temperature difference is $\vartheta_{l2} - \vartheta_{l1} = \text{const}$, then the quantity $\Delta T_{1,2}$ in the stationary mode will depend principally on λ_0:

$$\Delta T_{1,2} = \frac{A_1}{A_2 + \lambda_0}, \tag{7}$$

where A_1 and A_2 are quantities which can be considered constant for the pair of heat probes selected:

$$A_1 = \frac{8(\vartheta_{l2} - \vartheta_{l1})\lambda_m ks}{3\pi^2 R \operatorname{sh} kl}; \quad A_2 = \frac{8\lambda_m ks \operatorname{cth} kl}{3\pi^2 R}. \tag{8}$$

The independence of $\Delta T_{1,2}$ from U_0 indicates that under the assumptions mentioned above the temperature difference of the specimen surface at the probing points should vary with the change in specimen temperature to the degree by which it is influenced by the change in λ_0 which hence originates (see Fig. 1). In particular, for $\lambda_0 = \text{const}$ the temperature difference $\Delta T_{1,2}$ should remain constant with the change in U_0. This indicates the possibility of conducting measurements on specimens having a temperature which differs from the temperature of the ambient medium within certain limits, while the instrument can be calibrated at some one temperature, for instance, room temperature.

However, in practice the presence of heat exchange between the specimen and the ambient medium for $U_0 \neq 0$ and its associated nonuniformity of the specimen temperature field can affect the reading of the thermal comparator, and this influence will be more significant, the greater the absolute value of U_0. In addition, in practice it is difficult to eliminate convective and radiative heat exchange between the specimen and the heat probes outside the contact limits. The change in intensity of this heat exchange with the change in U_0 is equivalent to a diminution of augmentation of the effective radius of contact R, which affects the recorded temperature difference $\Delta T_{1,2}$, as is seen from (7) and (8).

It should be noted that it was also assumed in deriving (7) that the distortions of the specimen temperature field induced by each heat probe are not superposed for any λ_0. However, in practice this condition cannot be satisfied for a fixed spacing between the heat probes. Moreover, the magnitude of the radius of contact R, just as the other parameters in (8), cannot always be determined with the accuracy needed. Consequently, the method being

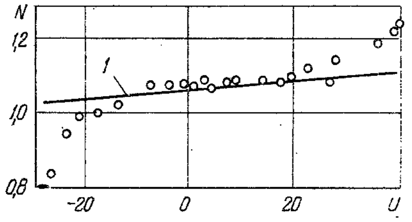

Fig. 2

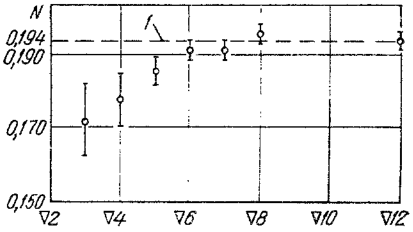

Fig. 3

Fig. 2. Experimental data on measurement of the thermal conductivity of K-8 glass at various specimen temperatures different from room temperature: 1) standard dependence of the coefficient of thermal conductivity of K-8 glass on the temperature according to the results in [5]; U, °C.

Fig. 3. Experimental results of multiple measurements of the thermal conductivity of organic glass at 20°C in specimens with different roughness of the surface being probed: 1) standard value of the coefficient of thermal conductivity of organic glass at 20°C according to the results in [5].

described cannot be used as absolute and requires preliminary calibration on standard thermal-conductivity specimens.

According to (7), the sensitivity of the method

$$S_0 = \frac{\Delta(\Delta T_{1,2})}{(\Delta\lambda_0/\lambda_0)}$$

will be a maximum under the condition

$$A_2 = \lambda_0, \tag{9}$$

on whose basis the optimal heat-probe parameters and the radius of their contact with the specimen surface are selected by using (8).

As heat probes, we used cable thermocouples consisting of Chromel–Copel thermoelectrodes of 0.1 mm diameter pulled through stainless steel tubes of 1 mm outer diameter. At a 10 mm spacing from the junctions, found by means of (9) for λ_0 = 1 W/(m·°K), the cable thermocouples were soldered to the copper plates 3 of 1.6 mm thickness which were pressed to the "cold" and "hot" surfaces of the TBM-24 type thermal battery 4. A constant temperature difference ($\approx$40°K) was maintained on the copper plates by using a Chromel–Copel differential thermocouple 5 and an electronic regulator of the current through the thermal battery assembled according to the circuit presented in [4]. Regulation was by means of a null signal obtained by canceling the emf of the differential thermocouple 5 by a direct voltage of 2.5 mV from a dc source. The spacing between the cable thermocouples was determined by the size of the thermal battery and was 14 mm. The cable thermocouples were connected differentially and switched to a KSP-4 type potentiometer. In order to the able to make measurements on electrically conducting specimens the junction of one of the cable thermocouples was insulated electrically from the metal tube. Constancy of the thermal contact between the thermal probes and the specimens was assured because the ends of the thrmocouples were worked on a spherical surface and were pressed to the specimen with a constant force because of the intrinsic weight of the whole probe.

Specimens of organic glass, glass of the brands TF-1, K-8, LK-5, fused quartz of the brand KV, pure iron (99.95%), and copper, recommended in [5], were used as standard thermal-conductivity specimens to calibrate and investigate the thermal comparator produced. All the standard specimens had at least a class ∇10 for working the surface being probed. The interpolation dependence of $\Delta T_{1,2}$ on λ_0 obtained differs from (7) and is

$$\Delta T_{1,2} = \frac{A_1}{A_2 + \lambda_0} + A_3, \tag{10}$$

where A_1 = 17.0 W/m; A_2 = 1.02 W/(m·°K); A_3 = 6°K. The constant A_3 corresponds to the minimal temperature difference at the ends of the heat probes as $\lambda_0 \to \infty$, and its appearance is explained, in particular, by the temperature drop in the contact zone between the heat probes and the specimen and in the thermocouple junctions.

In conformity with (10), the potentiometer scale was calibrated directly in units of the thermal conductivity, which permitted direct measurement of λ. Measurements on a K-8 glass specimen, which has a different temperature at the room temperature of the surrounding air, showed that the thermal comparator yields correct results at a specimen temperature in the range −15 to 30°C (Fig. 2). Results when the systematic deviations from the standard values of the thermal conductivity of K-8 glass did not exceed the random root-mean-square error were taken as correct. The temperature of its surface, on which the probing was performed, was taken as the specimen temperature. The temperature drop over the specimen thickness of 7 mm hence varied between −7°K at a −15°C temperature to +1.5°K at the +30°C temperature.

The appearance of considerable systematic errors at temperatures above +30°C is explained by a sharp rise in the intensity of the radiative heat exchange between the specimen and the heat probes outside the limits of direct contact, which is equivalent to an increase in the radius of the contact circles between the heat probes and the specimen. The influence of the nonuniformity of the temperature field in the specimen at these temperatures is not felt, since the temperature drop over the specimen thickness, which exceeds 1.5°K in absolute value, was also observed at temperatures below +10°C at which the thermal comparator yields a correct result according to Fig. 2. At temperatures below −15°C, the lowered readings of the thermal comparator are explained principally by degradation of the thermal contact between the heat probes and the specimen because freezing of the water vapor from the air occurred on the specimen surface. Degradation of the thermal contact is equivalent to diminution of the radius of the effective contact circles between the heat probes and the specimen.

The additional error in measurement because of surface roughness becomes commensurate with the random error for a class of working the surface being probed above ∇6, as has been established by multiple measurement of λ in organic glass specimens with different surface treatment (Fig. 3).

Investigations of the influence of specimen thickness on the thermal comparator readings permitted establishment that the minimal specimen thickness depends slightly on λ_0 and is $\approx$ 5 mm for λ_0 = 0.04-2.0 W/(m·°K).

The error in determining λ in specimens satisfying the above-mentioned parameters by using the thermal comparator produced is 5% for λ_0 = 0.04-5.0 W/(m·°K), as experimental investigations showed, and is determined by the calibration error by using standard specimens and by the error of the individual measurement. The error rises for $\lambda_0 > 5.0$ W/(m·°K). However, it can be diminished by an appropriate selection of the thermal probes, by starting from the condition (9) for larger values of λ_0.

In conclusion, it should be noted that the thermal comparator developed can be used not only for express measurements of λ for poor heat conductors, but also for thermal defectoscopy, for example, to detect defects in the form of unglued parts in multilayered structures, as well as to determine the thickness of heat-insulated coatings on metal articles.

NOTATION

λ, coefficient of thermal conductivity; λ_m, coefficient of thermal conductivity of the heat probes; λ_0, coefficient of thermal conductivity of the specimen; α, coefficient of heat exchange between the heat probes and the ambient medium; p, s, l, perimeter, cross section, and length of the heat probes; $\vartheta(x)$, excess temperature of the heat probes relative to the ambient medium at a spacing x from the lower ends; ϑ_{01}, ϑ_{02}, excess temperatures of the lower ends of the heat probes making contact with the specimen; ϑl_1, ϑl_2, excess temperatures of the upper ends of the heat probes; U_0, excess temperatures of the specimen relative to the ambient medium; $U_{av}(R, 0)$, average excess temperature of the specimen surface in a circle of radius R; S_0, relative sensitivity of the method; $\Delta\lambda_0$, $\Delta(\Delta T_{1,2})$, increment in the specimen thermal conductivity and its corresponding increment in the temperature difference; $k = \sqrt{\alpha p/\lambda_m s}$; N, reading on the thermal comparator [W/(m·°K)].

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NONSTEADY METHOD OF DETERMINING HEAT FLUX

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A method is described for determining intense heat flux, based on solution of the linearized heat-conduction equations. The data obtained are compared with data determined by the steady calorimeter method and the quasisteady method.

In present practice intense heat flux is measured by a number of techniques which have certain defects as well as advantages.

For example, the steady types of calorimeters are typically of complex construction and have definite cooling limitations in large heat-flux conditions, i.e., it is impossible to eliminate the maximum heat rate in a short time interval (because of the properties of the calorimeter material). Therefore, nonsteady methods of measuring intense heat flux have been developed recently. In particular, it was proposed in [1] to measure heat flux using a sensor which is so short that the temperature difference between the front and back walls would be negligible. This assumption will be valid only for a thin-walled sensor [2]. Otherwise, measurement of a large heat flux can introduce considerable error. For a thin sensor the measurement of heat flux requires the use of high-speed recording equipment and materials to withstand a large heat load, because of the absence of heat removal.

In [3] it has been suggested to measure heat flux by the use of quasistationary heat conditions. It should be noted that the assumption of equality of heating rates on the forward and rear sensor walls is based on solution of the linear heat-conduction equation. This assumption breaks down if nonlinearity is taken into account. Experiments have also confirmed that the rate of heating of a body differs at each point. Therefore, first of all, this method is based on an *a priori* incorrect assumption and, therefore, contains an *a priori* inherent error. Secondly, for the formula $q = \sigma\rho C(dt/d\tau)$, on which the theory of this method is based, to apply, it is necessary that the temperatures and the heating rates be the same throughout the entire body. But if we assume that the heating rates are equal at all points of the body, as the quasistationary method suggests, the temperatures at the forward and rear walls will be quite different. Therefore, at these points there will also be different values of specific heat capacity, and this must be accounted for in determining the heat flux. However, this difference cannot be accounted for in the above equation. Therefore, there is an additional error in the method.

Several papers have proposed to determine heat flux using sensors containing several thermocouples. For example, a method was proposed in [4-6] for measuring heat flux using sensors containing four thermocouples. This method differs from those described above in

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